The Possibility of Micro-fission Chain-reactions and their Application to the Controlled Release of Thermonuclear Energy

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It had been shown 1,2 recently that it may be possible to reach densities up to $10^3\,\mathrm{g/cm^3}$ by imploding a solid hydrogen pellet with the aid of a symmetric arrangement of multiple laser beams irradiating the pellet simultaneously from all sides. The purpose of this proposed scheme was to reduce the energy input requirements for the controlled release of thermonuclear energy delivered by laser ignited T-D micro-explosion systems.

It is shown in this paper that by applying the foregoing concept of high density compression to a pellet consisting of fissionable material such as U235, Pu239, or U223, the critical mass for the initiation of a fission chain reaction in these materials can be reduced by many orders of magnitude raising the possibility of micro-fission-explosions with laser energy inputs for compression within the expected reach of laser technology. Alternatively, the pellet compression may be also achieved by the employment of intense relativistic electron beams. In addition, by surrounding the fissionable pellet with a layer of dense T-D material to be compressed together with the pellet, the minimum pellet size can be furthermore reduced by neutron reflection from the high neutron albedo T-D shell. For fissionable pellets without a reflector the critical mass can be as small as ~ 0.3 g requiring a laser energy input for compression of several megajoule. For a pellet surrounded by a T-D reflector the critical mass can be reduced down to $\sim 10^{-3}$ g with a laser energy input for compression of several 10^5 joule.

In addition to the neutron albedo effect of the T-D shell there is a bootstrap mechanism whereby the T-D mantle achieves thermonuclear temperatures resulting in the emission of thermonuclear reaction neutrons into the fissionable pellet causing more fission reactions and thereby raising the temperature of the pellet. This then in turn will lead to an additional heating of the T-D blanket resulting in an increased flux of thermonuclear neutrons raising the fission rate even further. Because of the greatly increased material density, a fission chain reaction initiated in it will grow in proportion much faster than in uncompressed fissionable material as it is being used in conventional fission bombs.

The system described may have important usefulness as a small fission power plant especially for space propulsion applications but also for a more effective burn in a thermonuclear micro-bomb reactor as a fission supported fusion reaction.

1. Introduction

There are two unconventional concepts for the controlled release of thermonuclear energy which have been proposed several years ago and which have the common goal to ignite a small thermonuclear pellet of solid densities such as liquid or solid T-D by heating it to fusion temperatures. Devices based on these unconventional concepts are also known as thermonuclear micro-bomb systems since they resemble on a micro-scale thermonuclear bombs except that the fission trigger is replaced by some other means. If successful, these systems may lead to the controlled release of thermonuclear energy as a promising alternative to the more conventional magnetic field confinement concept. In one of these systems 3 it is proposed to bombard and heat a T-D pellet by a giant pulsed laser beam. In another

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approach 4 it is proposed to achieve the same goal by the bombardment with an intense relativistic electron beam. In the laser beam method of bombardment the coupling in between the photons and the plasma target results from classical and nonclassical (turbulent) photon absorption. In the electron beam method the coupling relies on collective beam plasma instabilities. The results of calculations done in order to determine the minimum required energy input vary widely depending on a multitude of different assumptions. Such different assumptions for example are: different intrinsic laser efficiencies, the efficiency of energy deposition of the laser or electron beam into the thermonuclear target, whether the pellet is "tamped" by some high Z-material and whether a magnetic field either applied externally, or in case of an electron beam supplied by the beam itself, can quench the electronic heat conduction losses from the pellet to the tamp. In spite of the different assumptions made it is widely believed that input energies of at least 107 joule and perhaps by as



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This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License. much as 10⁹ joule would be required. Energies in the range of 10⁷ joule are within the reach of powerful electron beam generators but are difficult to attain with known laser systems. For energy inputs up to 10⁹ joule the technical problems are, of course, increased in proportion and maybe outside the realm of present day technology at least for laser beams. (In the case of electron beams, energies of 10⁹ joule may perhaps be possible by the employment of the magnetic field insulation principle ^{4, 5}.)

Because of the obvious technical difficulties resulting from the contemplated very large energy requirements, two ways have been proposed which may promise to reduce drastically the minimum energy input. One of these schemes applies to the electron beam method and suggests to employ the strong beam magnetic field, which can be as large as 108 gauss, to reduce the electron heat conduction losses and also quench the charged fusion products to within the reaction zone 6. If successful this would lead to a thousandfold reduction in the minimum energy input for an effective thermonuclear burn. In the other scheme, which applies to the laser method system, a symmetric arrangement of a multiple number of laser beams is proposed, bombarding the T-D pellet simultaneously from all sides 1, 2. By properly shaping the time behavior of these laser beams, an almost adiabatic compression of the pellet can be achieved, raising the pellet density by a factor up to 104. At these greatly elevated densities the pressure in the compressed pellet material has risen to $\sim 10^{12}$ atm $\cong 10^{18}$ dyn/cm². In case infrared gas lasers are used, as CO, or chemical lasers, a minimum of six beams would suffice. Lasers of this type have the additional advantage of high efficiencies in combination with high total energy outputs. Besides compressing the pellet, the laser pulse must effectively heat the pellet material to thermonuclear temperatures. For this purpose the infrared lasers may be unsuited and one would have to use rather an assembly of UV laser beams. In this case, however, a substantially larger number than only six laser beams would be needed. Furthermore, no efficient high energy UV laser is in sight to fulfill these requirements. For this reason substantial difficulties have still to be overcome before such a system can become technically feasible.

In the concept put forward here, it is proposed to primarily compress rather than heat a small pellet of fissionable material into a critical assembly for the initiation of a fast neutron chain reaction. Here then for the pellet compression the highly efficient high energy infrared gas lasers can be used combined with a comfortably small number of only six beams. In contrast to the UV lasers required for the thermonuclear fusion pellet concept the technology of high energy infrared gas lasers is already advanced to a very high degree of perfection.

Alternatively, the pellet may be bombarded and compressed by several intense relativistic electron beams. It seems conceivable that six such electron beams will suffice. Intense relativistic electron beams can be produced much more easily and inexpensively than laser beams at much higher total energy outputs. By magnetic self-confinement these beams can be also concentrated into very small areas which is required for our purpose ⁶. The one disadvantage of electron beams, if compared with laser beams, which is their longer pulse length could be circumvented by imploding a hollow shell rather than a solid pellet.

If furthermore the fissionable pellet is surrounded by a layer of dense thermonuclear T-D material, to be compressed together with the fissionable pellet to high densities, two things can be achieved simultaneously: 1) the dense thermonuclear T-D material will serve as an effective neutron reflector substantially reducing the critical radius and thus critical mass of the compressed fissionable pellet and 2) thermonuclear reactions in the T-D reflector will be greatly enhanced by the fission chain reaction in the pellet making possible the fission-supported controlled release of thermonuclear energy. The system resembles a miniaturized conventional hydrogen bomb with a fissionable trigger whereby the fissionable pellet will greatly reduce the laser energy input which would be normally required to ignite a T-D pellet without fission support. Furthermore, with the proposed fission-fusion micro-bomb system the energy delivered per exploding pellet is expected to be substantially larger than for a pure T-D pellet. In the laser T-D pellet system, as reported under Reference 1 and 2, a rather large number of pellets, for example 100, would have to be detonated per second in order to achieve an interesting power production level. Such a large number of pellet explosions per second raises obvious technological problems. In the fission supported pellet system the energy per pellet explosion for example can be 100 times larger thus reducing the numbers of pellets to 902 F. Winterberg

be exploded per second together with the technological problems accordingly.

2. Adiabatic Compression of a Fissionable Pellet with Fermi Degenerate Electron Distribution

In the reported theoretical models for laser pellet compression pressures up to 10^{12} atmospheres $\cong 10^{18}$ dyn/cm² are predicted by properly shaping the time behavior of the laser beams impinging onto the pellet. We will now discuss the consequences of such a pressure when applied to a pellet of fissionable material, for example, a pellet consisting of U235.

Because the ultrahigh pressure is caused by an ablation process of T-D material during its irradiation by the laser beams, it is not immediately obvious that the same pressure would result in case the pellet consists of a high A-number material such as uranium. Since the higher the ablation-product velocity the higher will be the implosion pressure, it is rather to be expected that a low A-number material with a higher ablation-product velocity will give rise to a larger pressure than for high A-number material. However, in case of high A-number pelletmaterial a high ablation-product velocity can still be achieved by simply surrounding the high A-number pellet material with a concentric layer of solid hydrogen of proper thickness. During the irradiation by the laser light only the hydrogen layer will be ablated and in its course be strongly compressed. The pressure in this hydrogen shell must, of course, reach the same order of magnitude as for the compression of a T-D pellet and which was estimated to be of the order of 10¹⁸ dyn/cm². This high pressure is then acting on the fissionable material which thereby is compressed to a much higher density than does exist under normal conditions. The maximum density is obtained for an adiabatic compression of the pellet material with a Fermi degenerate electron distribution. Under these optimal conditions matter can be described by the Thomas Fermi equation of state 7. For hydrogen this equation of state yields at a pressure of $p = 10^{18} \, \text{dyn/cm}^2$ an atomic number density of about 104 times larger than normal, that is $\sim 5 \times 10^{26} \, \text{cm}^{-3}$, corresponding to a density of $\sim 10^3 \,\mathrm{g/cm^3}$. If applied to uranium, the Thomas Fermi equation of state at a pressure of 1018 dyn/cm2 yields an atomic number density of about 240 times larger than normal, that is $\sim 10^{25}$ cm⁻³, corresponding to a density of 4.5×10^3 g/cm³. More precisely the values of the atomic number densities for hydrogen at normal pressure and at $10^{18}\,\mathrm{dyn/cm^2}$ are

$$N_0 = 5 \times 10^{22} \text{ cm}^{-3}$$
 and $N_1 = 5 \times 10^{26} \text{ cm}^{-3}$.

For U235, the corresponding values are

$$N_0 = 4.86 \times 10^{22} \ \mathrm{cm^{-3}}$$
 and $N_1 = 1.17 \times 10^{25} \ \mathrm{cm^{-3}}$.

These atomic number densities will be used for the following estimates.

3. The Critical Radius and Mass of an Unreflected Pellet of Fissionable Material

For the following calculations classical neutron diffusion theory will be used supplemented by the appropriate transport corrections at the pellet boundary 8 . In order to get a simple estimate a one group neutron diffusion model will suffice. We furthermore will omit the α -correction resulting from nonfission neutron absorption which is relatively insignificant at high neutron energies as it is the case here. An inclusion of the α -correction would leave our results essentially unchanged.

The critical radius R_0 of a spherical mass of fissionable material above of which a chain reaction will occur is then given by

$$R_0 = \pi/B_{\rm m} - d , \qquad (1)$$

where

$$B_{\rm m}^{\ 2} \cong 3 \ \sigma_{\rm s} \ \sigma_{\rm f} \ N^2(\nu - 1) \,,$$
 (2)

and

$$d \cong 0.71/N \,\sigma_{\rm s} \,. \tag{3}$$

In Eq. (2) and (3) $\sigma_{\rm s}$ and $\sigma_{\rm f}$ are the nuclear neutron cross sections for scattering and fission and ν is the average number of neutrons set free per fission reaction. N is the atomic number density of the fissionable material. The quantity $B_{\rm m}$ is called the material buckling, and d the extrapolation length. $B_{\rm m}$ determines the curvature of the neutron flux within the critical assembly. A large value of $B_{\rm m}$ implies a small critical radius. At the extrapolation length d the neutron flux computed from simple diffusion theory is put equal to zero. To make an estimate we will assume the following values:

$$\begin{split} \sigma_{\rm s} &\cong 2 \times 10^{-24} \ {\rm cm^2} \ , \\ \sigma_{\rm f} &\cong 2 \times 10^{-24} \ {\rm cm^2} \ , \\ \nu &\cong 2.9 \ , \\ N &= 1.17 \times 10^{25} \ {\rm cm^{-3}} \ . \end{split}$$

The values for the cross sections are approximately valid for fast neutrons released by fission reactions after a slight slowing down process being caused by inelastic collisions. The chosen atomic number density corresponds to a pressure of $10^{18}\,\mathrm{dyn/cm^2}$.

If the fissionable pellet is surrounded by a hydrogen blanket for the purpose of achieving the high ablation pressures considered, the neutrons can in part be also slowed down by elastic collisions with the hydrogen nuclei of the blanket. This will be, of course, also true if the fissionable pellet is surrounded by a layer of T-D material acting as a neutron reflector, a case which will be considered below.

With the assumed numerical values we then obtain

$$B_{\rm m} \cong 56 \, {\rm cm}^{-1}$$
 and $R_0 \cong 2.6 \times 10^{-2} \, {\rm cm}$.

The energy required to compress the pellet to a volume V under the pressure p is approximately.

$$E \cong \frac{2}{3} p V. \tag{4}$$

Inserting $V=(4~\pi/3)~R_0^{~3}=7.35\times 10^{-3}~{\rm cm^3}$ and $p=10^{18}~{\rm dyn/cm^2}$ results in

$$E = 4.9 \times 10^{13} \text{ erg} = 4.9 \text{ MJ}$$
.

Although this energy is rather large it may be attainable with infrared giant pulse gas lasers. Furthermore, the required energy can be reached with intense relativistic electron beams in case the pellet is bombarded and compressed by a number of such beams.

The critical mass for a uranium density of $\varrho \cong 4.5 \times 10^3 \, \mathrm{g/cm^3}$ is given by $m_0 = (4 \, \pi/3) \, \varrho \, R_0^{\, 3} = 0.34 \, \mathrm{g}$. Ordinary fission bombs are known to have a critical mass of several kilograms whereas the described micro-fission bomb has a critical mass $\sim 10^4$ times smaller. A pellet of fissionable material with a mass above this critical value can detonate in a micro-fission-explosion by a fast rising neutron chain reaction.

4. The Critical Radius and Mass for a Reflected Pellet of Fissionable Material

A further substantial reduction of the minimum critical mass is possible by surrounding the fissionable pellet with a layer of solid hydrogen acting as a neutron reflector and which is being compressed together with the pellet to high densities.

The neutron absorption cross section of D and T is small as compared to the other cross sections.

Therefore, if the neutron reflector consists of a layer of solid T-D, neutron absorption in it can be neglected for the dimensions under consideration. In order to obtain simple estimates we will make use of this approximation.

For a spherical fissionable pellet of radius R and reflector thickness T the critical equation is given by 8

$$\cot B_{\rm m} R = (1/B_{\rm m} R) (1 - D_{\rm r}/D_{\rm c}) - D_{\rm r}/D_{\rm c} B_{\rm m} T.$$
(5)

In Eq. (5) transport corrections for the extrapolated length are neglected since the T-D layer is compressed to such a large density as to make the scattering mean free path small compared to the other physical dimensions. D_c and D_r refer to the neutron diffusion coefficients in the fissionable core and reflector material respectively with

$$D_{\rm c} \cong 1/3 \, N_{\rm c} \, \sigma_{\rm cs} \,, \tag{6}$$

$$D_{\rm r} \cong 1/3 \, N_{\rm r} \, \sigma_{\rm rs} \,. \tag{7}$$

 $\sigma_{\rm cs}$ and $\sigma_{\rm rs}$ are the neutron scattering cross sections of the core and reflector material. Consistent with our order of magnitude estimates, we will assume that $\sigma_{\rm cs} \cong \sigma_{\rm rs} \cong 2 \times 10^{-24} \, {\rm cm}^2$, which is in reasonably good agreement with the scattering cross sections at the occuring high neutron energies. $N_{\rm c}$ and $N_{\rm r}$ are the atomic number densities of the core and the reflector. We will assume that both the reflector and the core are compressed by a pressure of $\cong 10^{18} \, {\rm dyn/cm}^2$ such that $N_{\rm c} \cong 1.17 \times 10^{25} \, {\rm cm}^{-3}$ and $N_{\rm r} = 5 \times 10^{16} \, {\rm cm}^{-3}$. We thus have $D_{\rm r}/D_{\rm c} \cong 2.3 \times 10^{-2}$.

Since for a strongly neutron reflected pellet $B_{\rm m} R$ is sufficiently small we can put

$$\cot B_{\rm m} R \cong 1/B_{\rm m} R - \frac{1}{3} B_{\rm m} R \tag{8}$$

and obtain after some rearrangement from Eq. (5) for the critical radius

$$R = \frac{3}{2} \frac{D_{\rm r}}{D_{\rm e} B_{\rm m}^2 T} \left[1 + \left(1 + \frac{4}{3} \frac{D_{\rm c}}{D_{\rm r}} B_{\rm m}^2 T^2 \right)^{1/2} \right]. \quad (9)$$

Putting in Eq. (9) $T \rightarrow \infty$, which corresponds to a thick reflector, one has asymptotically

$$R_{\rm min} = (1/B_{\rm m}) (3 D_{\rm r}/D_{\rm c})^{1/2} \cong 4.7 \times 10^{-3} \, {\rm cm}$$
.

A thick reflector means, for example, that

$$\frac{4}{3} (D_c/D_r) B_m^2 T^2 \cong 4$$
, (10)

as can be seen by inspection of Eq. (9). From Eq. (10) one then would obtain

$$T = (1/B_{\rm m}) (3 D_{\rm r}/D_{\rm c})^{1/2} \lesssim R$$
. (11)

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Making the reflector thickness larger would not more reduce the critical radius R appreciably. The smallest energy for compression is given if the sum of the core and reflector volume assume an absolute minimum. This minimum should be in the neighborhood where $R \cong T$.

The total energy required to compress both the fissionable pellet and the reflector under a pressure of $p \cong 10^{18} \, \text{dyn/cm}^2$ is determined by Eq. (4) with $V = (4 \, \pi/3) \, (R+T)^3$. One thus obtains

$$E \cong 2.3 \times 10^{12} \text{ erg} = 2.3 \times 10^5 \text{ J}$$
.

This value for the required compression energy is about 20 times smaller than for an unreflected fissionable pellet. The critical mass of the pellet of radius $R_{\rm min}$ and density $\varrho \cong 4.5 \times 10^3 \, {\rm g/cm^3}$ is given by $m_{\rm min} = (4\,\pi/3) \, \varrho \, R_{\rm min} \cong 1.9 \times 10^{-3} \, {\rm g}$, which is 180 times smaller than for an unreflected pellet.

5. The Initiation and Development of the Fission Chain Reaction

A neutron chain reaction in a pellet of critical dimension would be just self-sustaining such that the neutron gain by multiplication would be exactly compensated by neutron losses through the pellet surface. For a micro-fission-explosion, as it is envisioned here, the pellet dimension must be sufficiently well above its critical size, for example, with a pellet radius increased by 50%. Assuming such an increase in the pellet size the calculated compression energies and pellet masses will be larger by a factor $(1.5)^3 \cong 3.4$. The compression energies for an unreflected pellet would be thus raised to ~17 MJ and for a reflected pellet to ~ 1 MJ. These energies may still be attainable with high energy infrared lasers and are already within the technical feasibility of intense relativistic electron beams.

After the pellet has been compressed into a supercritical state, a neutron chain reaction will start according to

$$n = n_0 \exp\{N \,\sigma_{\rm f} \,v_0 \,(\nu - 1) \,t\}. \tag{12}$$

where n_0 is the initial number of neutrons and n the number of neutrons released after the time t. v_0 is the neutron velocity. With $N=1.17\times 10^{25}\,\mathrm{cm}^{-3}$, $\sigma_{\mathrm{f}}\cong 2\times 10^{-24}\,\mathrm{cm}^2$, $v_0\sim 10^9\,\mathrm{cm/sec}$, $v-1\sim 2$ one obtains

$$n \cong n_0 \exp\{4.7 \times 10^{10} t\}$$
. (13)

From Eq. (13) follows as the e-fold time $t_e \cong 2$ $\times 10^{-11}$ sec. The pellet disassembly time is $\tau \sim R/v$, with $v \leq 10^7 \, \text{cm/sec}$ the speed of sound in uranium at keV temperatures. Specifically for $R \cong 10^{-2}$ cm, $\tau \sim 10^{-9}$ sec, which is thus the minimum inertial confinement time. Therefore, if the pellet is confined for a period of $t \cong 10^{-9}$ sec, after which the chain reaction breaks off, one would have $n \cong n_0 e^{50} = 10^{22} n_0$. This would be more than sufficient to fission all the nuclei within the pellet. Because of the greatly increased atomic number density the fission chain reaction proceeds here at a much faster pace than in conventional fission bombs with fissionable material at normal densities. For a fast raising chain reaction a sufficiently large initial neutron number n_0 is required. In the reflected pellet these initial neutrons can be supplied by thermonuclear reactions in the T-D blanket heated to fusion temperatures during the shock compression by the laser pulse.

Finally, for a pellet with a T-D reflector there is a bootstrap mechanism which may substantially reduce the laser energy requirement for compression and also accelerate the explosive process considerably. This bootstrap mechanism results from the following coupling effect: If thermonuclear neutrons are produced in the T-D shell these neutrons will increase the fission rate in the fissionable pellet core. The increased fission rate in turn will lead to an increased heating of the T-D shell resulting in the production of even more thermonuclear neutrons which then will further increase the fission rate. One can therefore speak of a fusion assisted fission-chainreaction or in reverse of a fission-chain-reaction assisted fusion reaction. This, of course, is a process developed in nuclear weapons technology and must have been well studied.

6. Magnetic Target Compression

We would like to discuss here another compression method which applies both to laser and electron beams and which is an alternative to the hydrodynamic compression of the target. This alternative method is based on the possibility to produce by the electron or laser beam very strong magnetic fields by which a magnetic target compression can be achieved. The method is most easily explained for electron beams, and we will for this reason first discuss it in this case.

If an intense relativistic electron beam is projected into a tenuous background plasma it can carry a large self-magnetic field ⁶. The condition to be satisfied is essentially to choose the density of the background plasma sufficiently high as to compensate the electric space charge resulting in a slightly larger repulsive electric force of the beam electrons as opposed to the self-confining attractive magnetic force. The density of the background plasma at the other hand must be chosen sufficiently low such that the induced return currents in the background plasma cannot compensate the self-magnetic beam field. The radial self-electric and azimuthal self-magnetic beam fields are given by

$$E_{\rm r} = -2 \,\pi \, r \, e \, n_{\rm e} (1 - \varphi_{\rm e}),$$
 (14)

$$H_{\varphi} = -2 \pi r e n_{\rm e} (1 - \varphi_{\rm m}) v/c,$$
 (15)

where r is the beam radius, $n_{\rm e}$ the electronic number density in the beam. $\varphi_{\rm e}$ and $\varphi_{\rm m}$ are the degree of charge and current neutralization given by

$$\varphi_{\rm e} = n_{\rm i}^{(0)}/n_{\rm e} \,, \tag{16}$$

$$\varphi_{\rm m} = n_{\rm i}^{(0)} v_{\rm e}/n_{\rm e} v \cong n_{\rm i}^{(0)} v_{\rm e}/n_{\rm e} c = v_{\rm e} \varphi_{\rm e}/c$$
. (17)

In Eq. (16) and (17) $n_i^{(0)}$ is the ion number density of the background plasma with Z=1. Furthermore, $v_e < c$ is the drift-velocity of the backstreaming electrons which are contributing to the current neutralization; v is the drift-velocity of the beam electrons and it is assumed that $v \cong c$, which is fulfilled for intense relativistic electron beams. From the condition for beam equilibrium

$$E_{\rm r} = (v/c) H_{\omega} \tag{18}$$

it then follows that

$$(v/c)^{2} = (1 - \varphi_{e})/(1 - \varphi_{m}) \tag{19}$$

or
$$(v/c)^2 = (1 - \varphi_e)/(1 - v_e \varphi_e/c) < 1$$
. (20)

Since $1-\varphi_{\rm e}\!<\!1-v_{\rm e}\,\varphi_{\rm e}\!/c$ condition (20) can be fulfilled. Now if $n_{\rm i}^{(0)}\!\ll\!n_{\rm e}$ then

$$E_{\rm r} \cong -2 \pi r e n_{\rm e}, \qquad (21)$$

$$H_{\omega} \cong -2 \pi r e n_{e} v/c. \qquad (22)$$

Because of $I = \pi r^2 n_e e v$ one can also write instead off Eq. (22)

$$H_{\omega} = 2 I/c r = 0.2 I/r$$
, (23)

the latter being valid if *I* is expressed ampere representing the total electric current of the intense relativistic electron beam.

If the beam hits a small solid target of high electric conductivity such as a metallic pellet or a pellet surrounded by a plasma with a value of the conductivity $\sigma \cong 10^{18}\,\mathrm{sec}^{-1}$, the magnetic field will only penetrate a distance δ into the target which is equal to the skin depth

$$\delta = c \left(\tau / 4 \pi \sigma \right)^{1/2}, \tag{24}$$

where τ is the time during which the pellet is exposed to the magnetic field. Assume, for example, $\tau \sim 10^{-9}$ sec and it follows that $\delta \sim 10^{-4}$ cm. Since a total pellet radius $R+T\sim 10^{-2}$ cm was computed previously it follows that the magnetic field cannot penetrate into the pellet. As a result the magnetic field will compress the pellet.

In order to fulfill the required conditions of target compression one must have

$$H^2/8 \, \pi \cong 10^{18} \, \mathrm{dyn/cm^2}$$
 ,

or
$$H \cong 5 \times 10^9 \text{ gauss}$$
.

In the moment the beam hits the conducting target, the electric self-field of the beam will be cancelled, resulting in a strongly pinched beam surrounding the conducting target with an external beam radius of the same order of magnitude as the target radius. The needed beam current to produce the required magnetic field can thus be computed from Eq. (23) by substitution for r=R+T and one has

$$I \cong 5 (R+T) H. \tag{25}$$

Putting $R+T\cong 10^{-2}\,\mathrm{cm}$ and $H=5\times 10^9\,\mathrm{gauss}$ results in $I\cong 2.5\times 10^8\,\mathrm{ampere}$. This is a current not outside the realm of intense relativistic beam generators.

Next we discuss the magnetic compression idea in case of a laser beam. This is probably harder to achieve because it is more difficult to produce the required beams. Although in contrast to an electron beam, a laser beam does not have a static self-magnetic field, there is, nevertheless, the possibility that a laser beam of high intensity may produce a strong electron current within a plasma target by non-linear photon-electron interaction within the plasma medium $^{9, 10}$. According to these studies a coherent light beam with an average electric field $(E^2)^{1/2}$ produces an electron drift motion of plasma electrons in the forward direction of the beam with a drift velocity v_d given by

$$v_{\rm d} = e^2 \, \overline{E^2} / m^2 \, c \, \omega^2 \,.$$
 (26)

In Eq. (26) ω is the circular frequency of the beam electromagnetic wave; furthermore e and m are the electronic charge and mass. In order that this expression is valid it is required that $v_d \ll c$. For the light beam to excite this drift motion it is obvious that ω must be larger than the electron plasma frequency ω_p so that the wave can penetrate into the plasma, hence

$$\omega > \omega_{\rm p} = (4 \pi n e^2/m)^{1/2}$$
 (27)

or
$$v = \omega/2 \pi > v_p = 8.97 \times 10^3 \, n^{1/2} \, \text{sec}^{-1}$$
. (28)

In Eq. (28) n is the electron number density of the plasma. If, for example, $n=5\times 10^{22}\,\mathrm{cm^{-3}}$, which is the case for solid hydrogen, it follows that $v>2\times 10^{15}\,\mathrm{sec^{-1}}$ which corresponds to a wavelength of the laser light $\lambda<1.5\times 10^{-5}\,\mathrm{cm}$. This is a wavelength in the far ultraviolet, which would require the employment of an ultraviolet laser. However, one can see that this does not seem to be necessary if one surrounds the target to be compressed by a plasma of sufficiently lower density. If, for example, the target is surrounded by a plasma with a density 100 times smaller, the required wavelength of the laser light can be 10 times larger, which puts us in the infrared.

In order to show that the effect is independent of the wavelength of the laser light, we have to go back to Eq. (26). Introducing the Poynting vector defined by

$$S = (c/4 \pi) \ \overline{E^2} = P/A \ ,$$
 (29)

where P is the power of the laser and A the cross section of the laser beam, we obtain

$$v_{\rm d} = (e/m c)^2 (4 \pi P/A).$$
 (30)

J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmermann, Nature London 239, 139 [1972].

² K. A. Brueckner, KMS Fusion Inc., preprint, Ann Arbor, Michigan, April 1972.

⁴ F. Winterberg, Phys. Rev. 174, 212 [1968].

In Eq. (30) P/A is in erg/cm² sec. If we express P/A in Watt/cm² we obtain from Eq. (30)

$$v_{\rm d} = 3.9 \times 10^{22} \, P/\omega^2 \, A$$
. (31)

If, for example, $P/A=10^{15}\,\mathrm{watt}$, which is within the expectation of laser technology, furthermore $A=10^{-2}\,\mathrm{cm^2}$ and $\omega=10^{15}\,\mathrm{sec^{-1}}$, with $\lambda=1.9\times10^{-4}\,\mathrm{cm}$, which is in the near infrared, one has $v_\mathrm{d}=3.9\times10^9\,\mathrm{cm/sec}$, such that $v_\mathrm{d} \leqslant c$.

With the help of Eq. (31) we can now obtain a value for the total excited electron drift current given by $I = n e v_d A$. Expressing I in ampere one has

$$I = 6.25 \times 10^3 \, n \, P/\omega^2$$
. (32)

Putting in Eq. (32) $\omega = \omega_p$ we obtain finally

$$I = 1.97 \times 10^{-6} P$$
. (33)

It follows that the total electric current to be induced by this effect is independent of the plasma density. It is therefore possible to create the high magnetic field by surrounding the pellet to be compressed by a plasma of properly chosen density satisfying inequality (27). If, for example, an infrared laser is used, the density has to be of the order of $n \cong 10^{20} \, \mathrm{cm}^{-3}$. We take again our example with $P = 10^{15} \, \mathrm{watt}$ and obtain $I \cong 2 \times 10^9 \, \mathrm{ampere}$. If the plasma, surrounding the pellet of radius $R + T \cong 10^{-2} \, \mathrm{cm}$, has a radius of $r = 10^{-1} \, \mathrm{cm}$ it follows that the induced current will give rise to a magnetic field $H = 0.2 \, I/r = 4 \times 10^9 \, \mathrm{gauss}$, which is of the right magnitude to create a magnetic pressure of the order $10^{18} \, \mathrm{dyn/cm}^2$.

Finally, we would like to remark that the magnetic compression method can be, of course, also applied to a purely thermonuclear target to attain high target compression prior to thermonuclear ignition.

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